Harmonic Motion: Pendulums
Student Advanced Version

In this lab you will set up a pendulum using rulers, string, and small weights and measure how different variables affect the period of the pendulum. You will also use the concept of resonance to make pendulums swing without any initial push.

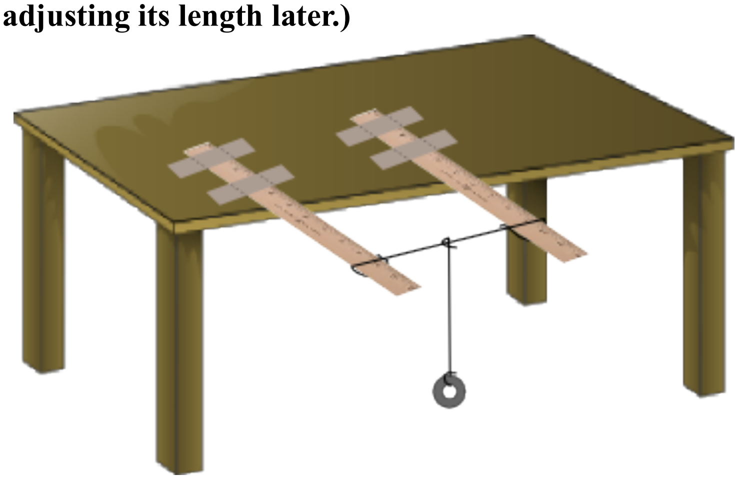
Key Concepts:

- The ***period*** is the amount of time it takes the pendulum to make one back-and-forth swing.
- The ***frequency*** is the number of swings the pendulum makes per unit time.
- The ***amplitude*** measures how far the pendulum is pulled back and swings.
- ***Resonance*** is the back-and-forth motion that becomes especially strong when a pendulum is repeatedly pushed at its natural frequency (the same frequency it already has as determined by the pendulum's length and the strength of gravity). This effect can happen with any system that has a natural vibrating frequency (e.g., earthquake toppling a building, opera singer breaking a crystal glass, etc.).

Part 1 – Period of a Pendulum

In this first part, you will investigate which factors affect the period of a pendulum.

1. If you're using duct tape, tape down two rulers to the table about 6 inches apart so that several inches stick out beyond the edge of the table as shown in the diagram below. If you're using masking tape instead, tape two rulers to opposite sides of the Tupperware container at the same height so that several inches stick out beyond the edge of the container. Place the box-ruler setup on the edge of the table such that the rulers stick out beyond the edge of the table.
2. Cut a piece of string about 30 cm long and tie it between the two rulers, making sure that it is pulled taut.
3. Cut a piece of string about 40 cm long and tie a washer to one end, then tie the other end to the string between the rulers. If you're using duct tape, your pendulum setup will look like the picture below. **(Do not tie the pendulum very tightly to the horizontal string – you will be adjusting its length later.)**

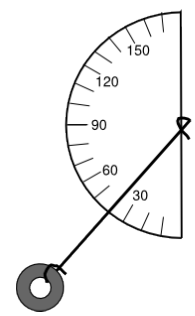


Effect of amplitude:

<u>Prediction</u>

*Q1. If we **increase the amplitude** of the pendulum's swing by pulling it further out, will the period of the pendulum (i.e., how long it takes to complete one swing) increase, decrease, or stay the same?*

<u>Measurement</u>



4. Using a protractor, pull out the pendulum to 20 degrees from vertical. Let it go and have your lab partner carefully time how long it takes for the pendulum to swing back and forth ten times. It can help to have one person count the complete swings out loud while another tracks the time. Use this time to calculate the period (the time for one back-and-forth swing).

5. Repeat, this time pulling out the pendulum to 40 degrees from the vertical.

Amplitude	**Time for 10 swings (seconds)**	**Period (seconds)**
20 degrees		
40 degrees		

Q2. Based on your experimental results, how does increasing the amplitude of the swing affect the period of the pendulum?

Effect of length:

<u>Prediction</u>

*Q3. If we **decrease the length** of the pendulum by shortening the string, will the period of the pendulum increase, decrease, or stay the same?*

<u>Measurement</u>

1. Copy over the data for the pendulum with three washers from the table on the previous page. **This will be your longer pendulum.**

2. Adjust the length of the pendulum with one washer so that it is about half as long as the other pendulum. Measure the time it takes for this shorter pendulum to swing back and forth 10 times. Use this time to calculate the period.

Length	Time for 10 swings (seconds)	Period (seconds)
longer pendulum		
shorter pendulum		

Q4. Why is it acceptable in this case to have two variables be different at the same time (length and number of washers) if we're interested in isolating the effect of the pendulum's length on its period?

Q5. Based on your experimental results, how does shortening the pendulum change the period of the pendulum?

The ***frequency*** of a pendulum is how many swings it can complete per second. Mathematically:

$$frequency = 1 \div period$$

Q6. Calculate the frequency of the two different-length pendulums.

Frequency of longer pendulum: ________________________________

Frequency of shorter pendulum: ________________________________

Q7. Which has the higher frequency, the short or the long pendulum?

Effect of mass:

<u>Prediction</u>

Q8. If we ***make the pendulum heavier*** *by attaching extra washers, will the period of the pendulum increase, decrease, or stay the same?*

<u>Measurement</u>

3. Copy the data from the first row of table on the previous page (in the "Effect of amplitude" section) into the first row of the table below.

4. Cut another piece of string and tie three washers to one end. Tie this new pendulum to the string between the rulers **next to your old pendulum.** Very carefully, adjust the length of the new pendulum so that the two are exactly equal. Remember, we only want to test one variable at a time!
5. Measure the time it takes for the heavier pendulum to swing back and forth ten times. Use this time to calculate the period.

Number of washers	Time for 10 swings (seconds)	Period (seconds)
1		
3		

Q9. Based on your experimental results, how does increasing the mass of the pendulum affect the period of the pendulum?

Q10. When the pendulum is made 2 times longer (from 20 cm to 40 cm), the period is changed by approximately a factor of:

(a) 2 *(b) 1/2* *(c)* $\sqrt{2} \approx 1.4$ *(d)* $1/\sqrt{2} \approx 0.7$

Optional Calculations:

Q11. Which of the following equations for the period of the pendulum is consistent with the trends you have discovered so far? Why?

Note: T is the period; g is the acceleration caused by gravity (higher g → stronger gravity); L is the length of the pendulum; m is the mass of the pendulum.

(a) $T = 2\pi \frac{L}{g}$ *(b)* $T = 2\pi g\sqrt{L}$ *(c)* $T = 2\pi \sqrt{\frac{g}{L}}$ *(d)* $T = 2\pi \sqrt{\frac{L}{g}}$

Q12. ***Challenge question:*** *How do we know the equation is not* $T = 2\pi \frac{\sqrt{L}}{g}$*?*

Hint: consider the units for g, L, and T. For the equation to be reasonable, the right hand side must give you the correct units for the period.

11. Use the equation you selected to predict the period of a pendulum with 1 washer that is 25 cm long. The acceleration of gravity is $g = 980 \text{ cm}/\text{s}^2$.

    Predicted period = ____________________

12. Adjust the length of the short pendulum in the middle to be 25 cm long. Move the two other pendulums aside. Measure the period of the 25cm pendulum and calculate how close you are to the prediction.

    Measured period = ____________________

    % error = (measured – predicted)/predicted x 100% = ___________________

Concept Questions

Q13. There is one other factor whose effect we did not measure: the strength of gravity. However, we can imagine running this experiment on the moon. When you pull out the pendulum, do you expect it to fall slower, faster, or with the same speed as on earth? What would happen to the period of the pendulum on the moon?

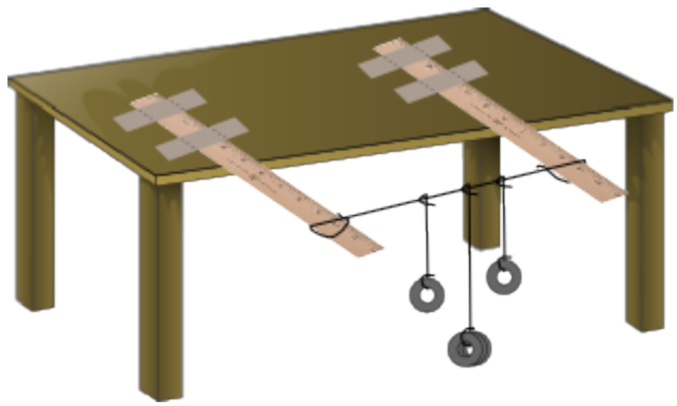
Q14. Suppose that a little 6-year-old and his big burly 13-year-old brother are swinging side-by-side on a playground. The strong 13-year-old pumps much harder and swings much higher than the 6-year old. Which of them takes longer to complete one back-and-forth motion?

Q15. If you are pushing a kid on a swing, do either you or the kid have any control over how quickly the swing swings back and forth? What aspect of the motion do you have control over?

Part 2 – Resonance

Any object that has a natural frequency for moving back and forth can experience ***resonance*** if it is pushed by a force that happens to match that frequency. When resonance occurs, the amplitude of the object's motion can become quite large. (If you were trying to push a friend on a swing to make them go as high as possible, you would push every time the swing comes back towards you – in other words, at the same frequency as the swing.) Without resonance, even a big force will not do much to keep the object moving. In this part, you will observe resonance in pendulums.

Haunted pendulums: Make a pendulum move without touching it!



1. Create another pendulum with one washer. Attach it to the string across the rulers, on the other side of the long pendulum (see diagram above). Carefully adjust the length of your new pendulum to be as close as possible to the length of the other short one.

2. Pull back one of the short pendulums and let it swing.

   Q16. What happens to the other short pendulum?

   Q17. What happens to the central long pendulum?

   Q18. Since you didn't touch it, where does the force that makes the other short pendulum swing come from?

   Q19. For which of the two pendulums you didn't touch does ***resonance*** *occur? Why did it happen for that one?*

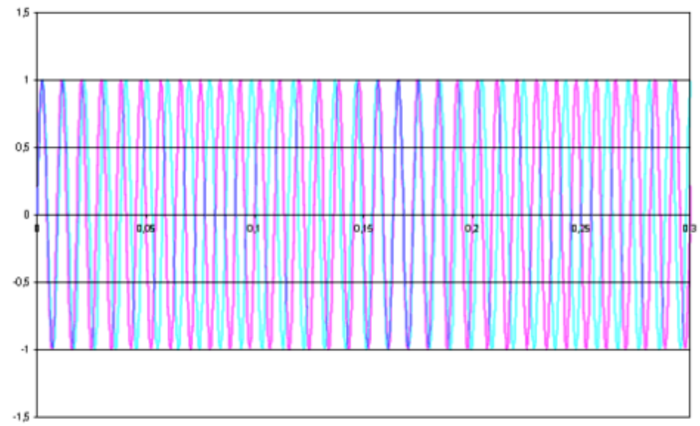
Q20. If you have two pendulums of the same length but with a different number of washers at the end, do you expect to see resonance?

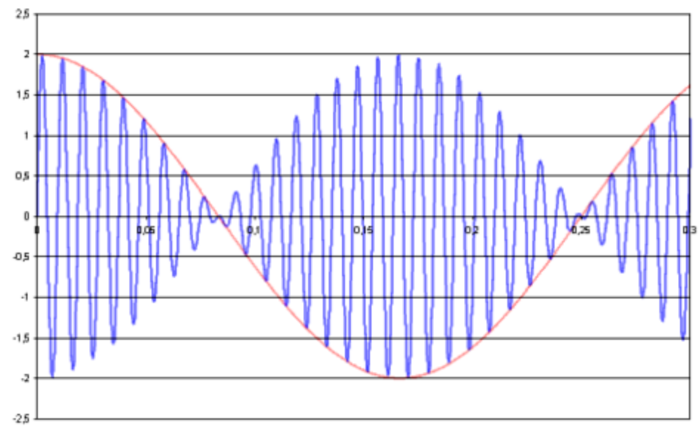
3. Now move the central long pendulum out of the way (by draping it over a ruler or the table). Repeat the experiment by pulling back one of the short pendulums and releasing it.

   Q21. If you wait a little longer, what happens to the pendulum you pulled back?

   Q22. On the axes below, make a rough plot of how the original pendulum that you pulled swings over time. Angle 0 corresponds to the pendulum hanging straight down. Positive angles are with the pendulum pointing towards you; negative angles are with the pendulum pointing away from you. Time 0 is when you first let go of the pendulum.

This phenomenon is called ***beats***. It happens when you have two different frequencies contributing to a back-and-forth motion. You can hear it with sound if you play two violin strings that are very close to the same frequency but not quite – the sound will seem to waver between loud and soft, in the same way that these pendulums alternated between large amplitudes and small ones. This phenomenon occurs because when two waves have different frequencies, they interfere with each other (see diagram below). Sometimes, the waves line up so that the interference is said to be ***constructive*** (making the net wave bigger than either of the individual waves); other times, the waves oppose each other so that the interference is said to be ***destructive*** (making the net wave smaller than either of the individual waves). Waves are all around us: waves on vibrating strings, ocean waves, sound waves, etc.





4. [OPTIONAL connection to sound] A great demonstration of sound beats can be done using an online "tuning fork" that will play a perfect pitch:

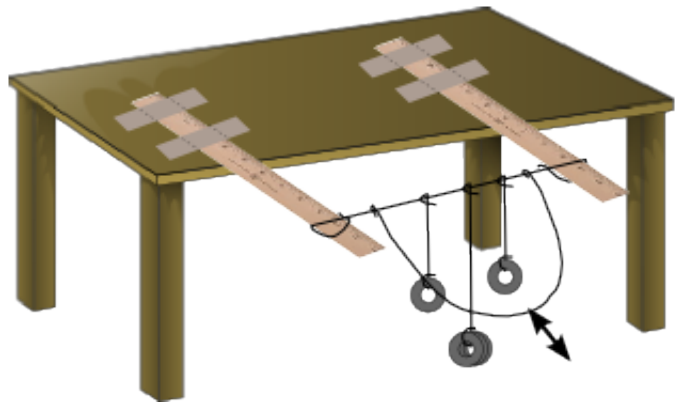
   http://www.tictone.com/online-tuning-fork.php

   Open up two browser windows such that both are playing the "A4" pitch at 440 Hz. Now adjust one of the pitches up by a couple Hz (i.e., to 442 Hz). You will hear beats.

   Q23. If you increase the difference between the pitches of the two simultaneous notes, how does this affect the frequency of the beats?

Pendulum "Mind-Control": Learn to control which pendulums swing!

5. Now attach a "handle" string at both ends to either side of the string between the rulers. Your pendulum setup will now look as shown below.



6. Tug rhythmically on the center of your handle-string.

   Q24. Can you make the two short pendulums swing high without the long pendulum swinging much?

   Q25. Can you make the long pendulum swing high without the short ones swinging much?

   Q26. Can you make only one of the short pendulums swing without the other one swinging?

   Q27. What do you have to change in order to control which pendulums are swinging?

Concept Questions

Q28. Old-fashioned grandfather clocks use a long pendulum that swings back and forth to keep time. When the metal of the pendulum is heated slightly, it stretches so that the pendulum becomes longer. On a warm summer day, is a grandfather clock likely to run fast, run slow, or keep good time?

Q29. Imagine a giraffe, a mouse, and an ant walking side-by-side. For every step the giraffe takes, the mouse will take many more, and an ant will take even more. In general, why do animals with short legs tend to move them more frequently than animals with long legs?

Q30. There have been several times in history when groups of soldiers marching in rhythm across a suspension bridge caused the bridge to start swinging and collapse. Nowadays, soldiers are ordered to break step when going across a bridge. Why is this? Why are the bridges unlikely to collapse from crowds of ordinary people walking across?

Q31. When designing a tall building in a region where earthquakes can happen, engineers have to take into account the typical frequencies associated with the back-and-forth vibration of the ground in an earthquake. Why?