Harmonic Motion: Pendulums
Teacher Advanced Version

In this lab you will set up a pendulum using rulers, string, and small weights and measure how different variables affect the period of the pendulum. You will also use the concept of resonance to make pendulums swing without any initial push.

Prerequisites:
Students doing the advanced version of this lab should be comfortable with arithmetic, square roots, and approximate plotting of an observation. The optional calculations section requires familiarity with algebraic expressions, and an optional challenge question requires basic dimensional analysis (units of physical quantities).

California Science Content Standards:
- **1. Newton's laws predict the motion of most objects.**
- 1a. Students know how to solve problems that involve constant speed and average speed.
- 1b. Students know that when forces are balanced, no acceleration occurs; thus an object continues to move at a constant speed or stays at rest (Newton's first law).
- 1c. Students know how to apply the law F=ma to solve one-dimensional motion problems that involve constant forces (Newton's second law).

Complete List of Materials:
For each group (2-3 students per group) you will need the following:
- 2 rulers or sticks and a flat table to which they can be taped
- 1 meter stick (or another ruler with cm markings)
- several meters of string or yarn
- 5 small weights (e.g., washers) that can be tied to the end of a string
- Duct tape OR masking tape
    - If you decide to use masking tape, you will also need a Tupperware container or equivalent approximately six inches in width to which rulers can be taped on opposite sides
- 1 protractor
- 1 stop watch
- OPTIONAL: a computer with Internet access for demonstration of "beats" with sound waves

Key Concepts:
- The ***period*** is the amount of time it takes the pendulum to make one back-&-forth swing.
- The ***frequency*** is the number of swings the pendulum makes per unit time.
- The ***amplitude*** measures how far the pendulum is pulled back and swings.
- ***Resonance*** is the back-and-forth motion that becomes especially strong when a pendulum is repeatedly pushed at its natural frequency (the same frequency it already has as determined by the pendulum's length and the strength of gravity). This effect can happen with any system that has a natural vibrating frequency (e.g., earthquake toppling a building, opera singer breaking a crystal glass, etc.).

Introductory Mini-Lecture:

In this lab we're going to do some experiments with pendulums. A pendulum is a long stick or string that has a weight attached on the bottom (the "bob") and is held fixed at one end so that it can swing back and forth. Actually, that is the most basic version of a pendulum, but there are plenty of more complicated systems in the real world that behave very similarly. Can you think of some real-world examples of things that swing back and forth like a pendulum? (Some examples include the pendulum in a grandfather clock, a swing on a playground, a chandelier, a suspension bridge, and even your arms and legs as you walk.)

We use two terms to describe how quickly a pendulum swings back and forth. The ***period*** of a pendulum is the length of time it takes the pendulum to make one full swing. The ***frequency*** is the inverse – how many swings the pendulum makes per unit time. These two terms can apply to any motion that repeats over and over again – so we can talk about the frequency of a sound wave, the period of the earth going around the sun, or the frequency of a Ferris wheel making one full rotation. So what would be the period of the earth turning all the way around its axis (from sunrise to sunrise)? (Answer: 1 day). What about the frequency of you brushing your teeth? (Answer: two per day, ideally.)

The first person to study the motion of pendulums was a famous physicist named Galileo. Over 400 years ago, Galileo was very interested in the motion of a chandelier he saw hanging from the ceiling. He noticed that as the chandelier swung back and forth, it did so very regularly – so regularly, in fact, that it could be used as a sort of clock. At his suggestion, a doctor friend of his even started using a small pendulum to time his patients' pulse. Galileo did several experiments to determine what factors affect how quickly a pendulum swings. Today, you're going to repeat some of these experiments by changing one variable at a time and measuring how the period of the pendulum changes. What are some variables that you think might affect how quickly it swings back and forth?

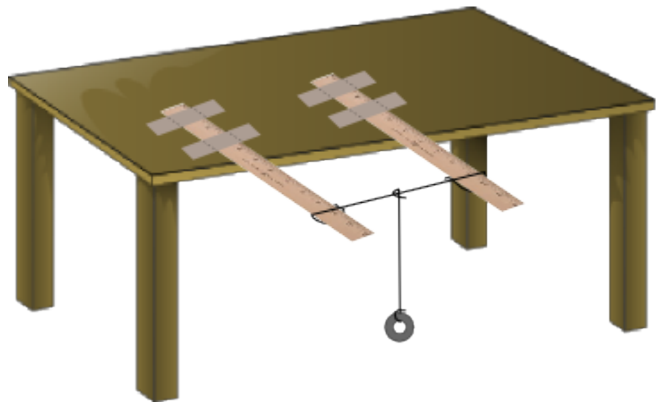
We're going to test the amplitude (how high the pendulum is swinging), the mass of the bob, and the length of the pendulum.

Part 1 – Period of a Pendulum

In this first part, you will investigate which factors affect the period of a pendulum.

1. If you're using duct tape, tape down two rulers to the table about 6 inches apart so that several inches stick out beyond the edge of the table as shown in the diagram below. If you're using masking tape instead, tape two rulers to opposite sides of the Tupperware container at the same height so that several inches stick out beyond the edge of the container. Place the box-ruler setup on the edge of the table such that the rulers stick out beyond the edge of the table.
2. Cut a piece of string about 30 cm long and tie it between the two rulers, making sure that it is pulled taut.

3. Cut a piece of string about 40 cm long and tie a washer to one end, then tie the other end to the string between the rulers. If you're using duct tape, your pendulum setup will look like the picture below. **(Do not tie the pendulum very tightly to the horizontal string – you will be adjusting its length later.)**



Effect of amplitude:

Prediction

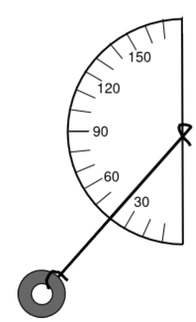
*Q1. If we **increase the amplitude** of the pendulum's swing by pulling it further out, will the period of the pendulum (i.e., how long it takes to complete one swing) increase, decrease, or stay the same?*

Discussion points:

- *One could argue that a pendulum pulled further out will have a longer distance to travel to make one full swing. By this reasoning, increasing the amplitude should give a longer period.*
- *One could also argue that if the pendulum starts higher up, it will be moving faster by the time it swings all the way down, in the same way that an object falling off a high building will be moving faster when it hits the ground than an object falling off a table. By this reasoning, increasing the amplitude should give a shorter period.*

Measurement

4. Using a protractor, pull out the pendulum to 20 degrees from vertical. Let it go and have your lab partner carefully time how long it takes for the pendulum to swing back and forth ten times. It can help to have one person count the complete swings out loud while another tracks the time. Use this time to calculate the period (the time for one back-and-forth swing).



   Note: Students simply need to divide the time for the pendulum to take 10 back-and-forth swings by 10 to calculate the period. If students aren't already familiar with the shortcut of moving the decimal point one place to the left to divide by 10, this would be a good time to teach them.

5. Repeat, this time pulling out the pendulum to 40 degrees from the vertical.

   Q2. Based on your experimental results, how does increasing the amplitude of the swing affect the period of the pendulum?

   The period of the pendulum should stay approximately the same regardless of amplitude.

   Discussion points:

   - *Notice that the pendulum never goes any higher than the height to which you originally pull it back. This means that if you hold the pendulum to your nose and let it go, it should never hit you on the nose (assuming you don't move your head and don't push the pendulum when you drop it). Try it and see!*
   - *After some time, the pendulum swing will become smaller and smaller. Why is this? This decrease in amplitude is called damping and is caused mostly by friction between the tied strings. In a perfect world, the pendulum would swing forever. In the real world, however, any pendulum will eventually stop unless it is pushed again.*

Amplitude	Time for 10 swings (seconds)	Period (seconds)
20 degrees		
40 degrees		

Effect of length:

<u>Prediction</u>

Q3. If we ***decrease the length*** *of the pendulum by shortening the string, will the period of the pendulum increase, decrease, or stay the same?*

Discussion points:

- *Ask students which pendulum has farther to go in order to make a full swing, the longer pendulum or the shorter one.*
- *Based on their answer, ask students which pendulum should take more time to make the swing.*
- *Most students should predict that the longer pendulum has farther to go and thus will take more time to make the swing.*

<u>Measurement</u>

6. Copy over the data for the pendulum with three washers from the table on the previous page. **This will be your longer pendulum.**

7. Adjust the length of the pendulum with one washer so that it is about half as long as the other pendulum. Measure the time it takes for this shorter pendulum to swing back and forth 10 times. Use this time to calculate the period.

Length	Time for 10 swings (seconds)	Period (seconds)
longer pendulum	*Copied from previous table*	*Copied from previous table*
shorter pendulum		

Q4. Why is it acceptable in this case to have two variables be different at the same time (length and number of washers) if we're interested in isolating the effect of the pendulum's length on its period?

Even though we're changing both the number of washers and the pendulum length for this experiment, we can still determine how the pendulum length affects the period because we know from the previous experiment that the number of washers doesn't affect the period.

Q5. Based on your experimental results, how does shortening the pendulum change the period of the pendulum?

A shorter pendulum should have a shorter period.

The ***frequency*** of a pendulum is how many swings it can complete per second. Mathematically:

$$frequency = 1 \div period$$

Q6. Calculate the frequency of the two different-length pendulums.

Frequency of longer pendulum: ____1 ÷ period of longer pendulum (from table above)____
Frequency of shorter pendulum: ____1 ÷ period of shorter pendulum (from table above)___

Q7. Which has the higher frequency, the short or the long pendulum?

The shorter pendulum has a higher frequency. Note that frequency and period are inversely related, so a pendulum with a long period will have a short frequency and vice-versa.

Effect of mass:

Prediction

Q8. If we ***make the pendulum heavier*** *by attaching extra washers, will the period of the pendulum increase, decrease, or stay the same?*

Discussion points:

- *Do heavy objects fall faster than lighter objects? (If applicable, make a connection to the* *gravity lab here!)*

- *If the students think that heavier objects fall faster, they should expect the heavier pendulum to take less time for a full swing (meaning it will have a shorter period).*
- *If the students think that heavier objects fall slower because they are harder to move, they should expect the heavier pendulum to have a longer period.*
- *Make sure the length of the pendulum remains unchanged after changing the number of washers. As discovered in the last section, the length of the pendulum will have an effect on the period.*

<u>Measurement</u>

8. Copy the data from the first row of table on the previous page (in the "Effect of amplitude" section) into the first row of the table below.

9. Cut another piece of string and tie three washers to one end. Tie this new pendulum to the string between the rulers **next to your old pendulum.** Very carefully, adjust the length of the new pendulum so that the two are exactly equal. Remember, we only want to test one variable at a time!

10. Measure the time it takes for the heavier pendulum to swing back and forth ten times. Use this time to calculate the period.

Number of washers	**Time for 10 swings (seconds)**	**Period (seconds)**
1	*Copied from previous table*	*Copied from previous table*
3		

Q9. Based on your experimental results, how does increasing the mass of the pendulum affect the period of the pendulum?

The period of the pendulum should stay about the same regardless of mass.

Discussion points:

- *Gravity pulls a heavier object more, which would tend to make the pendulum move faster, thus making the period shorter.*
- *On the other hand, a heavier object is also harder to move. This effect would tend to make the pendulum move slower, thus making the period shorter.*
- *In practice, these two effects cancel out so that heavy and light objects fall at the same rate. When we apply this principle to this pendulum problem, it makes sense that the mass of the pendulum doesn't affect the period.*

Teacher note: Students simply need to divide 1 by the period of each pendulum. For example, if the period of the longer pendulum is 1.3 seconds, then its frequency is 1 ÷ 1.3 = 0.77 swings per second.

Q10. When the pendulum is made 2 times longer (from 20 cm to 40 cm), the period is changed by approximately a factor of:

(a) 2 *(b) 1/2* *(c)* $\sqrt{2} \approx 1.4$ *(d)* $1/\sqrt{2} \approx 0.7$

Teacher note: To answer this question, students should compare their calculated periods from the table on the previous page.

Optional Calculations:

(For students who are taking of have taken a quantitative physical science class and are comfortable with algebraic manipulation; these should definitely be required for any student obtaining credit for a physics class.)

Q11. Which of the following equations for the period of the pendulum is consistent with the trends you have discovered so far? Why?

Note: T is the period; g is the acceleration caused by gravity (higher g → stronger gravity); L is the length of the pendulum; m is the mass of the pendulum.

(a) $T = 2\pi \frac{L}{g}$ *(b)* $T = 2\pi g\sqrt{L}$ *(c)* $T = 2\pi\sqrt{\frac{g}{L}}$ *(d)* $T = 2\pi\sqrt{\frac{L}{g}}$

We know that stronger gravity should make the period shorter, so only choices a and d could work. But we also know that if we double the length, we do not double the period, but only multiply it by the square root of 2. This means the length has to be within a square root.

*Q12. **Challenge question:** How do we know the equation is not* $T = 2\pi\frac{\sqrt{L}}{g}$*?*

Hint: consider the units for g, L, and T. For the equation to be reasonable, the right hand side must give you the correct units for the period.

Units of gravitational acceleration (g) are cm/s^2*. Units of length L are cm. The suggested equation would give units of √(cm)/(cm/s*^{2}*) = s*^{2}*/√(cm) which does not work out to the desired units for T (seconds). Note that choice (d) above does give the right units.*

11. Use the equation you selected to predict the period of a pendulum with 1 washer that is 25 cm long. The acceleration of gravity is $g = 980\ \text{cm/s}^2$.

$$T = 2\pi\sqrt{\frac{25}{980}} \approx 1.00\ sec$$

Predicted period = _____1.00 sec______

12. Adjust the length of the short pendulum in the middle to be 25 cm long. Move the two other pendulums aside. Measure the period of the 25cm pendulum and calculate how close you are to the prediction.

Measured period = ___________________

% error = (measured – predicted)/predicted x 100% = __________________

Concept Questions

Q13. There is one other factor whose effect we did not measure: the strength of gravity. However, we can imagine running this experiment on the moon. When you pull out the pendulum, do you expect it to fall slower, faster, or with the same speed as on earth? What would happen to the period of the pendulum on the moon?

On the moon, we would expect the pendulum to fall slower after we let it go, so it would take longer to complete the full back-and-forth motion. The weaker the gravity, the longer the period.

Q14. Suppose that a little 6-year-old and his big burly 13-year-old brother are swinging side-by-side on a playground. The strong 13-year-old pumps much harder and swings much higher than the 6-year old. Which of them takes longer to complete one back-and-forth motion?

Both of them will take the same time to swing back and forth! The period of the pendulum doesn't depend on how heavy the kid at the bottom is or on how high it swings, only on the length of the swing.

Q15. If you are pushing a kid on a swing, do either you or the kid have any control over how quickly the swing swings back and forth? What aspect of the motion do you have control over?

Neither of you can control how quickly the swing goes back and forth. That frequency is set entirely by the length of the swing. You can control the amplitude (how high they swing), however.

Introductory Mini-Lecture for Part 2:

Imagine that you want to push a child on a swing. We learned in the previous part that the swing acts like a pendulum and has a natural frequency for swinging back and forth, depending on how long it is. How would you push the swing to make it go higher and higher? You don't just pull it up once and then let go. Instead, you keep pushing every time the swing comes back towards you. In fact, you have to push with the exact same frequency as the swing – for every swing back and forth you make one pushing motion. If you were to push with a different, random frequency (imagine just flailing your arms around wildly), you would not have much luck in getting the swing to go high.

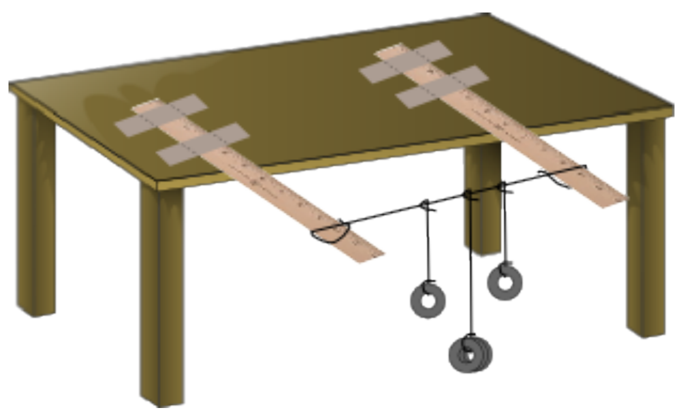
This effect is called **resonance**. In general, an object that has a natural frequency will move with much larger amplitude if it is pushed by a force with the same frequency. This applies not only to pendulums but to pretty much any regular motion. For instance, the crystal in a goblet has a natural frequency of vibration, and a really good opera singer can sing a note of exactly the right frequency to make it start vibrating and shatter. When designing a suspension bridge (like the Golden Gate Bridge), engineers have to take care that its natural swinging frequencies don't match up to any frequency it is likely to encounter in a wind storm (bridges have collapsed before due to resonance!).

In this part of the lab, we will observe resonance among pendulums.

Part 2 – Resonance

Any object that has a natural frequency for moving back and forth can experience ***resonance*** if it is pushed by a force that happens to match that frequency. When resonance occurs, the amplitude of the object's motion can become quite large. (If you were trying to push a friend on a swing to make them go as high as possible, you would push every time the swing comes back towards you – in other words, at the same frequency as the swing.) Without resonance, even a big force will not do much to keep the object moving. In this part, you will observe resonance in pendulums.

Haunted pendulums: Make a pendulum move without touching it!



1. Create another pendulum with one washer. Attach it to the string across the rulers, on the other side of the long pendulum (see diagram above). Carefully adjust the length of your new pendulum to be as close as possible to the length of the other short one.

2. Pull back one of the short pendulums and let it swing.

   Q16. What happens to the other short pendulum?

   It should start swinging as well, at the same natural frequency as the first short pendulum.

   Q17. What happens to the central long pendulum?

   It should not swing much if at all.

Q18. Since you didn't touch it, where does the force that makes the other short pendulum swing come from?

The original pendulum swinging back and forth transmits a force through the horizontal string. This force has the same frequency as the first pendulum's swing, which causes the other short pendulum to swing at that same natural frequency.

Q19. For which of the two pendulums you didn't touch does ***resonance*** *occur? Why did it happen for that one?*

The other short pendulum will resonate with the one you pulled back because it has the same natural frequency as the force coming through the horizontal string from the pulled pendulum. It has the same natural frequency as that force because it has the same natural frequency as the original short pendulum whose swinging caused that force to be transmitted through the horizontal string.

Q20. If you have two pendulums of the same length but with a different number of washers at the end, do you expect to see resonance?

Yes. All that matters is that their swinging frequency is the same, and we already showed that this frequency does not depend on the number of washers.

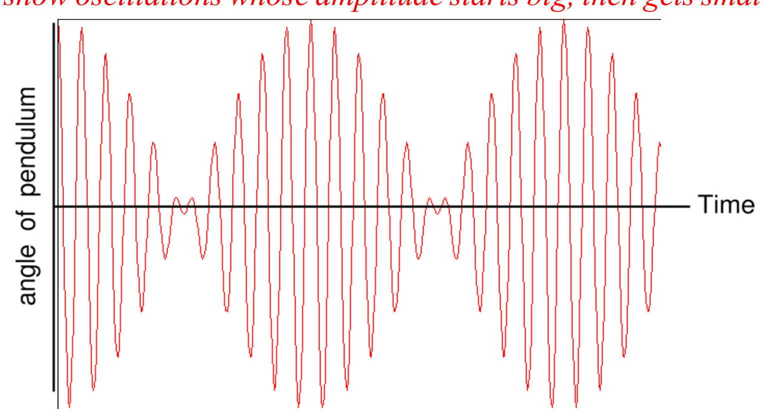
3. Now move the central long pendulum out of the way (by draping it over a ruler or the table). Repeat the experiment by pulling back one of the short pendulums and releasing it.

Q21. If you wait a little longer, what happens to the pendulum you pulled back?

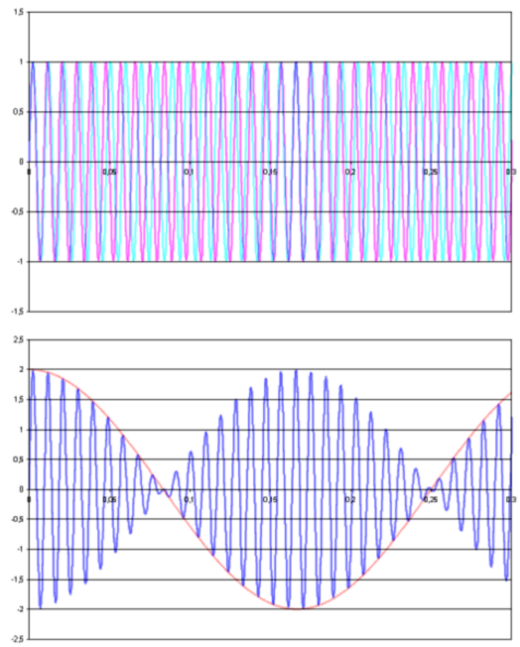
The amplitude of the pendulum you originally pulled back will get smaller and smaller until the pendulum almost stops swinging, then get larger and larger again as the second pendulum's amplitude becomes smaller and smaller. The two pendulums will continue to switch back and forth.

Q22. On the axes below, make a rough plot of how the original pendulum that you pulled swings over time. Angle 0 corresponds to the pendulum hanging straight down. Positive angles are with the pendulum pointing towards you; negative angles are with the pendulum pointing away from you. Time 0 is when you first let go of the pendulum.

The plot should show oscillations whose amplitude starts big, then gets smaller, then bigger again.



This phenomenon is called ***beats***. It happens when you have two different frequencies contributing to a back-and-forth motion. You can hear it with sound if you play two violin strings that are very close to the same frequency but not quite – the sound will seem to waver between loud and soft, in the same way that these pendulums alternated between large amplitudes and small ones. This phenomenon occurs because when two waves have different frequencies, they interfere with each other (see diagram below). Sometimes, the waves line up so that the interference is said to be ***constructive*** (making the net wave bigger than either of the individual waves); other times, the waves oppose each other so that the interference is said to be ***destructive*** (making the net wave smaller than either of the individual waves). Waves are all around us: waves on vibrating strings, ocean waves, sound waves, etc.



4. [OPTIONAL connection to sound] A great demonstration of sound beats can be done using an online "tuning fork" that will play a perfect pitch:

   http://www.tictone.com/online-tuning-fork.php

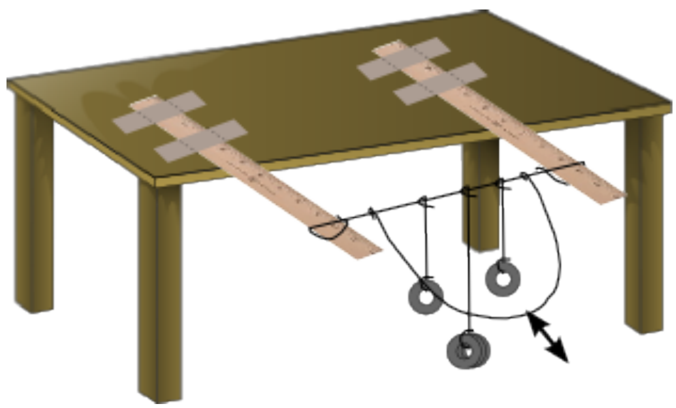
   Open up two browser windows such that both are playing the "A4" pitch at 440 Hz. Now adjust one of the pitches up by a couple Hz (i.e., to 442 Hz). You will hear beats.

   Q23. If you increase the difference between the pitches of the two simultaneous notes, how does this affect the frequency of the beats?

   The greater the difference in frequency between the two notes, the more frequent the beats.

Pendulum "Mind-Control": Learn to control which pendulums swing!

5. Now attach a "handle" string at both ends to either side of the string between the rulers. Your pendulum setup will now look as shown below.



6. Tug rhythmically on the center of your handle-string.

Note: This actually works better without consciously trying to pull at a certain frequency. If the students have difficulty getting this to work, suggest they imagine that they're pushing a child on a swing and focus on making the pendulum of desired length swing higher and higher. Also, it is easier if all the pendulums start at rest for each of the experiments.

Q24. Can you make the two short pendulums swing high without the long pendulum swinging much? Yes.

Q25. Can you make the long pendulum swing high without the short ones swinging much? Yes.

Q26. Can you make only one of the short pendulums swing without the other one swinging?

Students should not be able to do this if the pendulums are truly the same length because they have the same natural frequency.

Q27. What do you have to change in order to control which pendulums are swinging?

You have to change the frequency of pulling (how often you pull).

Concept Questions

Q28. Old-fashioned grandfather clocks use a long pendulum that swings back and forth to keep time. When the metal of the pendulum is heated slightly, it stretches so that the pendulum becomes longer. On a warm summer day, is a grandfather clock likely to run fast, run slow, or keep good time?

We know that longer pendulums swing more slowly (have longer periods), so the clock will run slow.

Q29. Imagine a giraffe, a mouse, and an ant walking side-by-side. For every step the giraffe takes, the mouse will take many more, and an ant will take even more. In general, why do animals with short legs tend to move them more frequently than animals with long legs?

While an animal is walking, its legs behave a little like pendulums, swinging back and forth under influence of gravity. Animals with shorter legs will tend to take faster steps because their legs have a faster natural frequency.

Q30. There have been several times in history when groups of soldiers marching in rhythm across a suspension bridge caused the bridge to start swinging and collapse. Nowadays, soldiers are ordered to break step when going across a bridge. Why is this? Why are the bridges unlikely to collapse from crowds of ordinary people walking across?

A bridge will collapse if the soldiers happen to be marching at the exact right frequency to create resonance with the natural frequencies of the bridge. Ordinary crowds of people walk at all different frequencies, so they won't generate any significant resonance.

Q31. When designing a tall building in a region where earthquakes can happen, engineers have to take into account the typical frequencies associated with the back-and-forth vibration of the ground in an earthquake. Why?

If the natural vibrating frequencies of the building happen to match the earthquake's frequency, the resulting resonance can make the building collapse.