Projectiles: Target Practice

Student Advanced Version

In this lab you will shoot a chopstick across the room with a rubber band and measure how different variables affect the distance it flies. You will use concepts of kinetic and potential energy to understand how the chopstick is propelled.

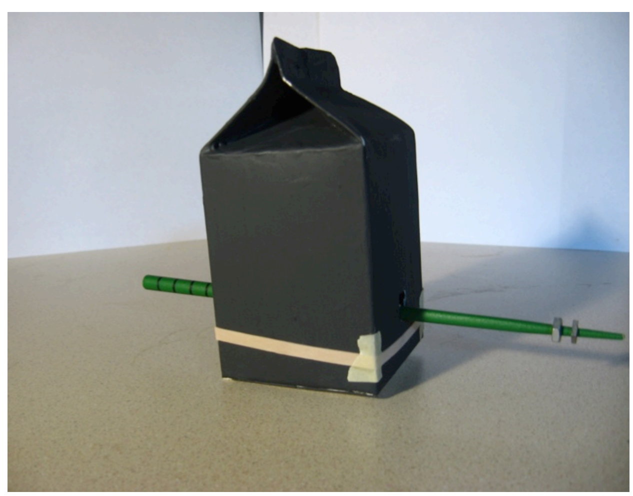
Key concepts:

- A ***projectile*** is a flying object given an initial push. The distance a projectile travels depends on how fast it flies and how much time it takes to hit the ground.
- Conservation of energy: When you stretch a rubber band, you store ***elastic potential energy***. The farther you stretch, the more elastic energy is stored. This can then be converted into ***kinetic energy*** (energy associated with motion) to make a projectile fly. The more kinetic energy it has, the faster it flies.
- The kinetic energy of an object of mass m, moving at velocity v is $KE = \frac{1}{2}mv^2$
- Physicists can come up with mathematical equations to describe real-world phenomena by using the concept of ***"proportional to"*** or ***"scales as."*** We say "x is proportional to y", or write "$x \propto y$", if x is equal to y times some constant.

Part 1 – Elastic energy

In this first part, you will look at how the distance traveled by the projectile is affected by how far back you pull the rubber band.

1. **Place a rubber band around the milk carton, as shown in the illustration to the right.** The rubber band should stretch below the hole from which the projectile will leave the carton, and on level with the hole in the carton through which the projectile will be pulled back to launch the chopstick. Do not readjust the rubber band after you start the experiment. **Put a little piece of tape at each of the two corners on the side of the carton through which the chopstick will leave to keep the rubber band in place.**



2. Orient the milk carton near the edge of the table such that it is pointed in a direction with at least 3 meters of open floor space. **Use a piece of masking tape to mark off the point of the floor in line with edge of the table from which the projectile will be launched.** Lay down the meter stick or tape measure on the line along which the chopstick will be launched. **If you're using a meter stick, mark off three 100-cm increments with masking tape to save yourself time later.**

3. **Put the projectile in the launcher as illustrated.** The projectile should be mostly horizontal when launched. The tail end of your projectile should be marked at 1 cm intervals**. Launch the projectile by pulling the rubber band and chopstick back to the 2 cm mark, then releasing the rubber band.** Make sure the nose of the projectile is lined up with the edge of the table when you launch. **One member of your group should stand to the side and watch where the tip of the projectile lands, while another should hold down the milk carton firmly on the table.** Repeat this process 2-3 times to ensure that you are launching the projectile consistently.

4. Once you are comfortable that you are launching the projectile consistently, pull the rubber band back to the 2 cm mark, release, and **measure the horizontal distance from the edge of the table to where your projectile landed.** Record it in the table below.

Data Table:

distance pulled (*d*)	**distance flown (*x*)**	**factor change (compared to 2 cm measurement)**
2 cm		1
3 cm		
4 cm		

5. Repeat steps 3 and 4, this time pulling back to the 3 cm and 4 cm mark.

6. In the last column of the table above, **fill in the factor change in distance flown when compared to the 2 cm distance.** To find the factor change, divide the distance x for each row by the distance in the first row. See the example table below for guidance on how to calculate the factor changes.

Example Table:

distance pulled (*d*)	**distance flown (*x*)**	**factor change (compared to 2 cm measurement)**
2 cm	75 cm	75 ÷ 75 = 1
3 cm	120 cm	120 ÷ 75 = 1.6
4 cm	145 cm	145 ÷ 75 = 1.93

Note: The numbers above are just example data points to demonstrate how the factor change calculations are done. The actual distance the chopstick flies will depend on many factors, including the type of rubber band used, the weight of the chopstick, and the height of the table. **Use the distances you recorded in the data table at the top of the page for your calculations, and fill in the answers in the last column of the data table.**

Q1. Which of the following best describes how the distance flown by the projectile (x) depends on how far you stretched the rubber band (d)? Hint: When you pulled back twice as far (to 4 cm versus 2 cm), how many times farther did the projectile fly?

(a) $x \propto d$ *(doubling the length pulled makes the projectile travel twice as far)*
(b) $x \propto d^2$ *(doubling the length pulled makes the projectile travel* $2^2 = 4$ *times as far)*
(c) $x \propto \sqrt{d}$ *(doubling length pulled makes the projectile travel* $\sqrt{2} = 1.4$ *times as far)*

Q2. Using your answer from the previous question, how far would you expect the projectile to travel if you pulled the rubber band back 5 cm?

Predicted distance traveled for d = 5 cm: x = _______________

7. Pull back 5 cm and **measure how far the projectile flies.**

Actual distance traveled for d = 5 cm: x = ________________

Q3. How close were you? Calculate the percent error:

$$\%\,\text{error} = \frac{(\text{actual distance - predicted distance})}{\text{predicted distance}} \times 100 = \underline{\qquad\qquad\qquad}$$

Concept Questions

Q4. Which of the following correctly describes what occurred when you pulled the rubber band back further to launch the projectile? (More than one choice may be correct!)

(a) More <u>elastic energy</u> was stored in the rubber band
(b) The projectile had more <u>kinetic energy</u> when it left the launcher
(c) The projectile took a longer time to hit the floor
(d) The projectile took a shorter time to hit the floor
(e) The projectile was moving faster when it left the launcher

The formula for elastic potential energy is: $PE = \frac{1}{2}kd^2$

The formula for the kinetic energy of an object of mass *m* moving at speed *v* is: $KE = \frac{1}{2}mv^2$

(*k* is the spring constant that gives the elastic strength of the rubber band, *d* is how far the rubber band is pulled back)

Q5. All of the kinetic energy of the projectile has to come from the stored elastic energy in the rubber band (the two energies must be equal). Solve for the speed of the projectile coming off the launcher (v) in terms of spring constant (k), distance pulled back (d), and mass of projectile (m).

Q6. What happens to the horizontal velocity of the projectile when you pull the rubber band back twice as far?

Part 2 – Projectile Mass

In this part, you will investigate the effect of the ***mass*** of your projectile on how far it travels. You will use a simple balance to estimate the projectile mass.

1. In your lab kit there should be a wooden bar with two Ziploc baggies attached. Use this balance as shown in the picture. When the masses in the two baggies are the same, the wooden bar should be horizontal.

2. Put your projectile into one baggie on your balance. If you hold the balance by the central string, it will tilt towards the heavier end, where you have placed the projectile.

3. **Add clay to the other baggie until the wooden bar is horizontal when you hold the balance by the central string.** The strings should line up with the black lines on the wooden beam. **Add this clay to the nose of the chopstick.**

Q7. By what factor has the chopstick mass changed?

4. Copy over your results from Part 1 into Column 2 of the table below. **Launch the projectile as before, pulling back to the 2 cm and 4 cm mark. Measure the distance traveled** and record it in Column 3 in the table below.

5. In Column 4, **fill in by what factor the distance changed when you altered the projectile mass.**

Column 1	Column 2	Column 3	Column 4
Distance pulled	**Distance traveled (original projectile- from original table)**	**Distance traveled (twice as heavy projectile)**	**Factor change** $(\boldsymbol{column\ 3 \div column\ 2})$
2 cm			
4 cm			

Q8. Which of the following best describes how the distance traveled by the projectile (x) depends on the mass of the projectile (m)?

(a) $x \propto m$ *(doubling the mass makes the projectile travel 2 times as far)*
(b) $x \propto \frac{1}{m}$ *(doubling the mass makes the projectile travel ½ times as far)*
(c) $x \propto \sqrt{m}$ *(doubling mass makes projectile travel* $\sqrt{2} = 1.4$ *times as far)*
(d) $x \propto \frac{1}{\sqrt{m}}$ *(doubling mass makes projectile travel* $\frac{1}{\sqrt{2}} = 0.7$ *times as far)*

Q9. How far do you think the new, heavy projectile will travel if you pull the rubber band to the

5 cm mark? (Hint: Recall what you learned in the previous section about the dependence of distance flown x on distance pulled back d).

Predicted distance when pulling 5 cm: ___________

6. **Place a target** at the predicted distance from the table. Now **launch the projectile** after pulling back 5 cm. **Did you hit the target?**

   Actual distance: ___________________

Concept Questions

*****For those who have done the gravity lab:

** Q10. Does making the projectile heavier change the time it takes to fall to the ground?*

The definition for kinetic energy is: $KE = \frac{1}{2}mv^2$

Q11. Using this formula and conservation of energy, explain why the heavier projectile did not fly as far. (Hint: Did the speed coming off the launcher change? Why?)

Part 3 – Starting Height

In this part, you will see how the distance that the projectile travels changes if you raise the launch height.

1. **Use a stack of textbooks to make the height from which you are shooting 1.5 times as big as it was before.**

   Original height of table: ________________
   New height of table: __________________ *(= original * 1.5)*

2. Copy over your results from Part 2 into Column 2 of the table below. **Shoot the projectile (with the modeling clay still on it)** as before, pulling back to the 2 cm and the 4 cm mark. Measure how far the projectile flies and fill in the following table. Column 4 is the factor change in the distance traveled when comparing the new and old height.

Column 1	Column 2	Column 3	Column 4
Distance pulled	**Distance traveled (original height, heavy projectile)**	**Distance traveled (new height, heavy projectile)**	**Factor change** $(\boldsymbol{column\ 3 \div column\ 2})$
2 cm			
4 cm			

Q12. Which of the following best describes how the distance traveled by the projectile (x) depends on launch height (h)?

(a) $x \propto h$ *(multiplying height by 1.5 changes distance traveled by a factor 1.5)*
(b) $x \propto h^2$
(multiplying height by 1.5 changes distance traveled by factor 1.5^2 *= 2.25)*
(c) $x \propto \sqrt{h}$
(multiplying height by 1.5 changes distance traveled by factor $\sqrt{1.5}$ *= 1.2)*
(d) $x \propto 1/h$
(multiplying the height by 1.5 changes distance traveled by 1/1.5 = 0.66)

*Q13. Using what you learned in part 2, how far do you think the projectile will fly if you launch from the new height, **take off the modeling clay**, and pull back 4 cm?*

*Predicted distance for **light projectile**, tall height, 4 cm:* ___________

3. **Place a target where you predict the projectile will land**. Make sure you have enough space so the projectile won't hit anything! **Did you hit the target?**

Concept Questions

Q14. Which of the following correctly describes what happens when you raise the height of the launcher?

(a) The projectile leaves the launcher with more kinetic energy.
(b) The projectile has a faster horizontal speed when leaving the launcher.
(c) The projectile takes longer to reach the ground
(d) More elastic energy is stored in the rubber band.

For an object falling under influence of gravity alone, the distance fallen (*h*) after a time *t* is given by, $h = \frac{1}{2}gt^2$, where g is the acceleration of gravity.

Q15. Solve for the time to hit the floor if starting from a table of height h (write an equation for t in terms of g and h).

Q16. What is the equation for the distance flown (x) in terms of the initial velocity of the projectile (v) and time of flight (t)?

Q17. Putting the above relationships together (solve for x in terms of g, v, h), do you see the same dependence of x on h as you noticed in your measurements (ex: $x \propto h$ *or* $x \propto h^2$ *or* $x \propto \sqrt{h}$*)?*

Wrap-Up Questions

Q18. If you change the following variables, what will happen to the horizontal distance flown by the projectile?

(a) *pull the rubber band back farther* ***increase / decrease / depends***

(b) *use a stronger rubber-band (one that takes more effort to pull)* ***increase / decrease / depends***

(c) *use a lighter projectile* ***increase / decrease / depends***

(d) *launch from a lower height* ***increase / decrease / depends***

(e) *launch on the moon (weaker gravity than earth)* ***increase / decrease / depends***

(f) *launch at an angle downwards* ***increase / decrease / depends***

(g) *launch at an angle upwards* ***increase / decrease / depends***
(Hint: Think about launching at a slight angle up, not launching straight upwards.)

Q19. Give two examples of projectile motion that you have seen in real life or on TV, and explain where the energy comes from to make the projectile move.

Q20. Putting together what you have learned in this lab about the dependence of the flight distance on different variables, which of the following best describes how the distance flown (x) varies with height (h), mass (m), and distance pulled back (d):

(a) $x \propto \frac{dh}{\sqrt{m}}$ *(b)* $x \propto d * \sqrt{\frac{h}{m}}$ *(c)* $x \propto d * \sqrt{\frac{m}{h}}$ *(d)* $x \propto h * \sqrt{\frac{d}{m}}$

Q21. If you pull the rubber band back twice as far, use a projectile that is 1/3 the mass, and launch from a height that is half as tall, by what factor will the distance flown by the projectile change?

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