

Projectiles: Target Practice

Teacher Advanced Version

In this lab you will shoot a chopstick across the room with a rubber band and measure how different variables affect the distance it flies. You will use concepts of kinetic and potential energy to understand how the chopstick is propelled.

Safety and Accommodation Notes:

- The projectile will always fly below chest height, but if there are multiple groups using the same shooting range, the students should still be warned to be careful and not hit someone with the projectile.
- Each group needs at least one student (or helper) capable of moving around to fetch the projectile and measure the distance to the targets. Predictions, weighing, and launching the projectile can be done without students needing to leave their chairs.

Prerequisites:

- Students doing the advanced version of this lab should be comfortable with basic algebra, proportionality, square roots, and arithmetic.

California Science Content Standards:

- **1. Motion and Forces: Newton's laws predict the motion of most objects.**
- 1d. Students know that when one object exerts a force on a second object, the second object always exerts a force of equal magnitude and in the opposite direction (Newton's third law).
- **1i: Students know how to solve two-dimensional trajectory problems.

Complete List of Materials:

For each group (2-4 students per group) you will need the following:

- 1 wooden chopstick
 - Mark the tail end at 1 cm intervals (up to 5 cm).
 - Test your projectile before running the lab. If the chopstick flies too far (i.e., if it hits a wall before hitting the ground), you can add a couple of metal nuts to the tip to reduce the distance the chopstick will fly.
- 1 pint-sized milk carton
- 1 pair of scissors or knife that you can use to make holes in the milk carton
- 1 meter stick or tape measure
- masking tape
- 1 balance, consisting of a wooden or cardboard bar with a hole at the center and two holes equidistant from the center, with a Ziploc bag attached to each edge hole (some wooden rulers already have holes punched for making such a balance)
- 1 rubber band (standard 5 mm size or similar)
- several grams of Play-Doh or modeling clay
- stack of textbooks or other props to increase the launch height
- a flat table or desk of reasonable height, with at least ~3 m free space in front of it
- ideally, 1 calculator per student; at a minimum, 1 per group

Preparation and Lab Notes:

To set up the milk carton as a base from which the chopstick will be launched, simply cut a hole in the center of one side of the carton, approximately 1 inch from the bottom edge. Make the hole large enough for the chopstick to slide through easily. Cut a similar hole in the center of the opposite face of the milk carton at as close to the same height as the first hole as possible. Test to make sure that the chopstick can slide easily through both holes and that it rests between the holes parallel to the ground.

Key concepts:

- A **projectile** is a flying object given an initial push. The distance a projectile travels depends on how fast it flies and how much time it takes to hit the ground.
- Conservation of energy: When you stretch a rubber band, you store **elastic potential energy**. The farther you stretch, the more elastic energy is stored. This can then be converted into **kinetic energy** (energy associated with motion) to make a projectile fly. The more kinetic energy it has, the faster it flies.
- The kinetic energy of an object of mass m , moving at velocity v is $KE = \frac{1}{2}mv^2$
- Physicists can come up with mathematical equations to describe real-world phenomena by using the concept of “**proportional to**” or “**scales as.**” We say “x is proportional to y”, or write “ $x \propto y$ ”, if x is equal to y times some constant.

Introductory Mini-Lecture:

In this lab we will study the motion of projectiles. A **projectile** is any object that is given an initial push and then flies by itself. For example, a cannonball shot out of a cannon is a projectile. So is a baseball thrown by a pitcher. Can you think of some other examples of projectiles? (Ex: bullet from a gun, arrow from a bow, spitball, rock from a slingshot)

Specifically, we’re going to be interested in how far a projectile flies before it hits the ground. Why does an arrow fly farther than a spitball and a bullet fly farther than a cannonball? In the lab, we will be using this launcher setup to launch a chopstick projectile with a rubber band (quick demo). What variables do you think will affect how far the chopstick flies? Remember, a variable is something we can play with to change the outcome of the experiment. (Some variables that students might come up are the strength of the rubber band, the height of the table, the size and weight of the chopstick, and how far the rubber band is pulled back.) We’re going to test three of the variables you suggested today.

To better understand what’s going on with a projectile, it is helpful to use the concept of **energy**. Energy is, more or less, the ability to make things move. A moving object has **kinetic energy** (which is why it can make other things move if it bumps into them). The faster an object moves, the more kinetic energy it has (a fast baseball can break a window, two cars colliding at high speed will suffer a lot more damage than slowly moving ones). Furthermore, a more massive object will also have more kinetic energy – so a cannonball would do a lot more damage than a BB pellet and an elephant can smash things more easily than a mouse. The formula for kinetic energy is $KE = \frac{1}{2}mv^2$.

Kinetic energy, the energy of moving objects, is not the only type of energy out there. Energy can also be stored in various forms as **potential energy**. There are many types of potential energy, and they can all be converted into motion (kinetic energy) if properly harnessed. In this lab, we're going to use **elastic potential energy**. We store energy in the rubber band by pulling it back. If we pull back a distance d , then the amount of energy stored is given by $PE = \frac{1}{2}kd^2$, where k is the "spring constant" that tells how strong the rubber band is. Then, when we let the rubber band go, this stored energy is converted into kinetic energy and the projectile starts moving. This is the basic principle behind launching any projectile. You always convert some stored energy (chemical energy in gunpowder to launch a bullet or in your muscles to throw a ball, elastic energy in bending a bow to launch an arrow) into kinetic energy of the projectile.

So what is it that determines how far a projectile flies? First of all, it definitely matters how much energy we give to the projectile. If I send it out moving faster (more kinetic energy) then it will fly farther. The other thing that matters is how much time the projectile has before stopping. It just keeps on flying forward until it hits the ground, and so the more time it has, the farther it will go.

****TEACHER NOTE****

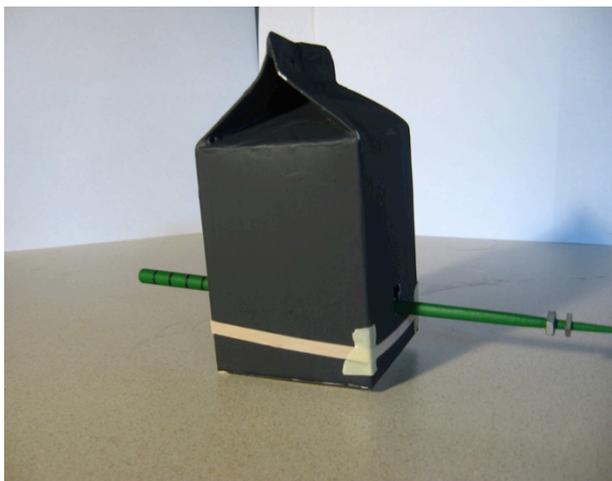
An instructional video for teachers can be found at https://www.youtube.com/watch?v=yf__ZMEjoXo

Part 1 – Elastic Energy

In this first part, you will look at how the distance traveled by the projectile is affected by how far back you pull the rubber band.

1. **Place a rubber band around the milk carton, as shown in the illustration to the right.**

The rubber band should stretch below the hole from which the projectile will leave the carton, and on level with the hole in the carton through which the projectile will be pulled back to launch the chopstick. Do not readjust the rubber band after you start the experiment. **Put a little piece of tape at each of the two corners on the side of the carton through which the chopstick will leave to keep the rubber band in place.**



2. Orient the milk carton near the edge of the table such that it is pointed in a direction with at least 3 meters of open floor space. **Use a piece of masking tape to mark off the point of the floor in line with edge of the table from which the projectile will be launched.** Lay down the meter stick or tape measure on the line along which the chopstick will be launched. **If you're using a meter stick, mark off three 100-cm increments with masking tape to save yourself time later.**
3. **Put the projectile in the launcher as illustrated.** The projectile should be mostly

horizontal when launched. The tail end of your projectile should be marked at 1 cm intervals. **Launch the projectile by pulling the rubber band and chopstick back to the 2 cm mark, then releasing the rubber band.** Make sure the nose of the projectile is lined up with the edge of the table when you launch. **One member of your group should stand to the side and watch where the tip of the projectile lands, while another should hold down the milk carton firmly on the table.** Repeat this process 2-3 times to ensure that you are launching the projectile consistently.

- Once you are comfortable that you are launching the projectile consistently, pull the rubber band back to the 2 cm mark, release, and **measure the horizontal distance from the edge of the table to where your projectile landed.** Record it in the table below.

Data Table:

Distance pulled (<i>d</i>)	Distance flown (<i>x</i>)	Factor change (compared to 2 cm measurement)
2 cm		1
3 cm		<i>Expected answer: about 1.5</i>
4 cm		<i>Expected answer: about 2</i>

- Repeat steps 3 and 4, this time **pulling back to the 3 cm and 4 cm mark.**

****TEACHER NOTE:**

To speed up the experiment, the students can simply mark off where the projectile ends on each trial with a piece of masking tape, and then do the measuring after they have done all three launches.

- In the last column of the table above, **fill in the factor change in distance flown when compared to the 2 cm distance.** To find the factor change, divide the distance *x* for each row by the distance in the first row. See the example table below for guidance on how to calculate the factor changes.

Example Table:

Distance pulled (<i>d</i>)	Distance flown (<i>x</i>)	Factor change (compared to 2 cm measurement)
2 cm	75 cm	$75 \div 75 = 1$
3 cm	120 cm	$120 \div 75 = 1.6$
4 cm	145 cm	$145 \div 75 = 1.93$

Note: The numbers above are just example data points to demonstrate how the factor change calculations are done. The actual distance the chopstick flies will depend on many factors, including the type of rubber band used, the weight of the chopstick, and the height of the table. **Use the distances you recorded in the data table at the top of the page for your calculations, and fill in the answers in the last column of the data table.**

Q1. Which of the following best describes how the distance flown by the projectile (x) depends on how far you stretched the rubber band (d)? Hint: When you pulled back twice as far (to 4 cm versus 2 cm), how many times farther did the projectile fly?

(a) $x \propto d$ (doubling the length pulled makes the projectile travel twice as far)

(b) $x \propto d^2$ (doubling the length pulled makes the projectile travel $2^2 = 4$ times as far)

(c) $x \propto \sqrt{d}$ (doubling length pulled makes the projectile travel $\sqrt{2} = 1.4$ times as far)

**TEACHER NOTE:

This is a good time to explain the concept of proportionality. We say “ y is proportional to x ”, or write “ $y \propto x$ ” if y is equal to something times x . (For example, the number of elbows in the room is proportional to the number of people in the room.) This means that if x is doubled, then y is doubled. If x is divided by 5 then y is divided by 5, and so on. You could also have a situation where y is proportional to the square of x ($y \propto x^2$) (ex: the number of tiles needed to cover a square floor is proportional to the length of the floor squared). This means that when x is doubled, then y is multiplied by $2^2 = 4$.

Q2. Using your answer from the previous question, how far would you expect the projectile to travel if you pulled the rubber band back 5 cm?

Predicted distance traveled for $d = 5$ cm: $x =$ _____

5 cm is 2.5 times longer than 2 cm. Since the distance traveled is proportional to distance pulled back, students can just multiply the x value for 2 cm by 2.5.

7. Pull back 5 cm and **measure how far the projectile flies.**

Actual distance traveled for $d = 5$ cm: $x =$ _____

Q3. How close were you? Calculate the percent error:

$$\% \text{ error} = \frac{(\text{actual distance} - \text{predicted distance})}{\text{predicted distance}} \times 100 = \underline{\hspace{2cm}}$$

Errors in the 5-25% range are common.

Concept Questions

Q4. Which of the following correctly describes what occurred when you pulled the rubber band back further to launch the projectile? (More than one choice may be correct!)

- (a) More elastic energy was stored in the rubber band
- (b) The projectile had more kinetic energy when it left the launcher
- (c) The projectile took a longer time to hit the floor
- (d) The projectile took a shorter time to hit the floor
- (e) The projectile was moving faster when it left the launcher

Pulling back the rubber band stores more elastic potential energy in the rubber. This gets converted to more kinetic energy for the projectile when it flies. More kinetic energy means the projectile is moving faster. The time to hit the floor depends only on the height of the table and the acceleration of gravity. It is not affected by the horizontal speed of the projectile or by how far you pull back the rubber band.

The formula for elastic potential energy is: $PE = \frac{1}{2}kd^2$

The formula for the kinetic energy of an object of mass m moving at speed v is: $KE = \frac{1}{2}mv^2$

(k is the spring constant that gives the elastic strength of the rubber band, d is how far the rubber band is pulled back)

Q5. All of the kinetic energy of the projectile has to come from the stored elastic energy in the rubber band (the two energies must be equal). Solve for the speed of the projectile coming off the launcher (v) in terms of spring constant (k), distance pulled back (d), and mass of projectile (m).

$$KE = PE \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}kd^2 \Rightarrow v = d \sqrt{\frac{k}{m}}$$

Q6. What happens to the horizontal velocity of the projectile when you pull the rubber band back twice as far?

If d is twice as big, the velocity (v) will be twice as big also.

Part 2 – Projectile Mass

In this part, you will investigate the effect of the *mass* of your projectile on how far it travels. You will use a simple balance to estimate the projectile mass.

1. In your lab kit there should be a wooden bar with two Ziploc baggies attached. Use this balance as shown in the picture. When the masses in the two baggies are the same, the wooden bar should be horizontal.
2. Put your projectile into one baggie on your balance. If you hold the balance by the central string, it will tilt towards the heavier end, where you have placed the projectile.
3. **Add clay to the other baggie until the wooden bar is horizontal when you hold the balance by the central string.** The strings should line up with the black lines on the wooden beam. **Add this clay to the nose of the chopstick.**



Q7. By what factor has the chopstick mass changed?

By adding enough clay to balance out the mass of the projectile to the nose of the chopstick, we have doubled the mass of our projectile. This corresponds to a factor of 2.

4. Copy over your results from Part 1 into Column 2 of the table below. **Launch the projectile as before, pulling back to the 2cm and 4 cm mark. Measure the distance traveled and record it in Column 3 in the table below.**
5. In Column 4, **fill in the distance factor change when you changed the projectile mass.**

Column 1	Column 2	Column 3	Column 4
Distance pulled	Distance traveled (original projectile-from original table)	Distance traveled (twice as heavy projectile)	Factor change (column 3 ÷ column 2)
2 cm			<i>Expected answer: about 0.7</i>
4 cm			<i>Expected answer: about 0.7</i>

Q8. Which of the following best describes how the distance traveled by the projectile (x) depends on the mass of the projectile (m)?

- $x \propto m$ (doubling the mass makes the projectile travel 2 times as far)
- $x \propto \frac{1}{m}$ (doubling the mass makes the projectile travel $\frac{1}{2}$ times as far)
- $x \propto \sqrt{m}$ (doubling mass makes projectile travel $\sqrt{2} = 1.4$ times as far)
- $x \propto \frac{1}{\sqrt{m}}$ (doubling mass makes projectile travel $\frac{1}{\sqrt{2}} = 0.7$ times as far)

Note: This is asking whether the numbers in column 4 are closest to 2, 0.5, 1.4, or 0.7.

Q9. How far do you think the new, heavy projectile will travel if you pull the rubber band

to the 5 cm mark? (Hint: Recall what you learned in the previous section about the dependence of distance flown x on distance pulled back d .)

Predicted distance when pulling 5 cm: _____

The answer is 2.5 x (distance flown by heavy projectile when pulled 2 cm).

6. **Place a target** at the predicted distance from the table. Now **launch the projectile** after pulling back 5 cm. **Did you hit the target?**

Actual distance: _____

Concept Questions

* For those who have done the gravity lab:

** TEACHER NOTE:

This question is not mandatory, as it draws more on concepts from the gravity lab. However, it is good to reinforce the idea to students that once the projectile is launched, the only force acting on the chopstick is gravity, which causes the chopstick to fall to the ground. This force of gravity is the same regardless of the horizontal force imparted to the chopstick when it is launched.

* Q10. Does making the projectile heavier change the time it takes to fall to the ground?

As demonstrated in the gravity lab (or as students may have seen in other science classes, the mass of an object does not affect how fast it falls. For students who are not familiar with this concept, now is a good time to discuss the classic experiment of dropping a tennis ball and a bowling ball off a roof – both will land at the same time.

To demonstrate, have a student stand on a chair and drop both a heavy and light projectile (with the clay tip pointing down to avoid it rotating in the air). Both should hit the ground at about the same time.

The definition for kinetic energy is: $KE = \frac{1}{2}mv^2$

Q11. Using this formula, explain why the heavier projectile did not fly as far. (Hint: Did its speed coming off the launcher change? Why?)

All of the projectile's energy has to come from the rubber band. If the projectile is heavier then it would need more energy to move at the same speed as the light projectile. However, we do not give it any more energy (we don't pull back any farther), and so it has to be moving slower when leaving the launcher. In the formula, the left-hand side stays the same, but m increases, so v has to decrease to make up for that. Since the projectile moves slower, it does not fly as far before hitting the floor.

It helps here to think of an extreme example – what if you were trying to shoot an elephant from the rubber band? The elephant would barely move. Its horizontal speed would be quite low. However, even a slowly moving elephant has quite a lot of energy (it could knock you over without needing to move fast).

Part 3 – Starting Height

In this part, you will see how the distance that the projectile travels changes if you raise the launch height.

1. Use a stack of textbooks to make the height from which you are shooting 1.5 times as big as it was before.

Original height of table: _____

New height of table: _____ (= original * 1.5)

2. Copy over your results from Part 2 into Column 2 of the table below. **Shoot the projectile (with the modeling clay still on it)** as before, pulling back to the 2 cm and the 4 cm mark. Measure how far the projectile flies and fill in the following table. Column 4 is the factor change in the distance traveled when comparing the new and old height.

Column 1	Column 2	Column 3	Column 4
Distance pulled	Distance traveled (original height, heavy projectile)	Distance traveled (new height, heavy projectile)	Factor change (column 3 ÷ column 2)
2 cm			
4 cm			

Q12. Which of the following best describes how the distance traveled by the projectile (x) depends on launch height (h)?

(a) $x \propto h$ (multiplying height by 1.5 changes distance traveled by a factor 1.5)

(b) $x \propto h^2$

(multiplying height by 1.5 changes distance traveled by factor $1.5^2 = 2.25$)

(c) $x \propto \sqrt{h}$

(multiplying height by 1.5 changes distance traveled by factor $\sqrt{1.5} = 1.2$)

(d) $x \propto 1/h$

(multiplying the height by 1.5 changes distance traveled by $1/1.5 = 0.66$)

****TEACHER NOTE:**

This just asks whether the factor changes are closest to 2, 0.5, 1.4, or 0.7.

Q13. Using what you learned in part 2, how far do you think the projectile will fly if you launch from the new height, **take off the modeling clay**, and pull back 4 cm?

Predicted distance for **light projectile**, tall height, 4 cm: _____

Start with the distance flown by the heavy projectile at the tall height, when pulled back 4 cm. To predict the distance for the light projectile in the same circumstances, just divide by 0.7 (or equivalently, multiply by $\sqrt{2} = 1.4$). Alternately, start with the distance flown by the light projectile at the short height when pulled back 4 cm (from Part 1) and multiply by $\sqrt{1.5} = 1.2$ to get the result for the new height.

3. **Place a target where you predict the projectile will land.** Make sure you have enough space so the projectile won't hit anything! **Did you hit the target?**

Concept Questions

Q14. Which of the following correctly describes what happens when you raise the height of the launcher?

- (a) The projectile leaves the launcher with more kinetic energy.
- (b) The projectile has a faster horizontal speed when leaving the launcher.
- (c) The projectile takes longer to reach the ground.
- (d) More elastic energy is stored in the rubber band.

Changing the height does not affect how fast the projectile flies horizontally, but it does give it more time before reaching the ground, so it can go further.

For an object falling under influence of gravity alone, the distance fallen (h) after a time t is given by, $h = \frac{1}{2}gt^2$, where g is the acceleration of gravity.

Q15. Solve for the time to hit the floor if starting from a table of height h (write an equation for t in terms of g and h).

$$t = \sqrt{\frac{2h}{g}}$$

Q16. What is the equation for the distance flown (x) in terms of the initial velocity of the projectile (v) and time of flight (t)?

$$x = v * t$$

(distance = rate * time)

Q17. Putting the above relationships together (solve for x in terms of g , v , h), do you see the same dependence of x on h as you noticed in your measurements (ex: $x \propto h$ or $x \propto h^2$ or $x \propto \sqrt{h}$)?

$$x = v * \sqrt{\frac{2h}{g}}$$

Hence, we see a square root relationship between height and distance, as should have been measured in the experiment.

Wrap-Up Questions

Q18. If you change the following variables, what will happen to the horizontal distance flown by the projectile?

The correct answers are underlined below, as well as the explanations behind them. This is a good time to revisit the concepts of kinetic and potential energy, as well as velocity, with the students.

(a) pull the rubber band back farther **increase** / decrease / depends

Pulling back further increases the amount of elastic energy stored in the rubber band. Thus the projectile is given more kinetic energy and it flies faster.

(b) use a stronger rubber-band (one that takes more effort to pull)
increase / decrease / depends

Using a stronger rubber band will also increase the amount of elastic energy stored for the same pulling distance.

(c) use a lighter projectile **increase** / decrease / depends

The amount of kinetic energy given to the projectile is the same. But the kinetic energy depends on both mass and speed. Thus, a lighter object will move faster if given the same kinetic energy, and the projectile will fly further.

(d) launch from a lower height **increase** / **decrease** / depends

The height does not change the speed with which the projectile is shot. However, a lower height means the projectile takes less time to hit the ground and thus it does not have time to fly as far.

(e) launch on the moon (weaker gravity than earth) **increase** / decrease / depends

If the effect of gravity is weaker, the projectile will not accelerate as quickly in the vertical direction. This means it will not be falling as fast, and thus will take longer to hit the ground. Since its horizontal velocity is unchanged, it can fly further in that longer time.

(f) launch at an angle downwards

increase / decrease / depends

Launching at an angle downwards gives the particle some vertical downwards velocity aside from what it acquires from gravity. It will thus fall faster, reach the ground quicker, and not have time to go as far.

(g) launch at an angle upwards

increase / decrease / depends

(Hint: Think about launching at a slight angle up, not launching straight upwards.)

If the projectile is launched slightly upward, then it will fly upwards for a while before turning and beginning to fall downwards. This means it will take a longer total time for it to hit the ground. Its total velocity remains unchanged (since the same amount of kinetic energy is given to it). However, some of that velocity goes into moving upward instead of forward. Thus the projectile has a lower horizontal speed but travels for a longer time. The change in the distance traveled depends on the exact angle. Launching straight up would maximize the time of travel but the projectile won't go anywhere. On the other hand, launching at a slight angle will increase the overall distance traveled. Actually, the maximum distance is achieved at a 45 degree angle. (** Students can experiment with this effect by threading the projectile through a different row of holes at the front of the basket.)

Q19. Give two examples of projectile motion that you have seen in real life or on TV, and explain where the energy comes from to make the projectile move.

Possible examples include:

Arrow shot from a bow (elastic energy from bending of the bow)

Blow-dart shot from a pipe (energy from the moving air you blow on it)

Cannon-ball (chemical energy stored in explosive gunpowder)

Bullet from a gun (same as above)

Thrown baseball (energy directly from your muscles, which can convert chemical into mechanical energy)

Q20. Putting together what you have learned in this lab about the dependence of the flight distance on different variables, which of the following best describes how the distance flown (x) varies with height (h), mass (m), and distance pulled back (d):

(a) $x \propto \frac{dh}{\sqrt{m}}$

(b) $x \propto d * \sqrt{\frac{h}{m}}$

(c) $x \propto d * \sqrt{\frac{m}{h}}$

(d) $x \propto h * \sqrt{\frac{d}{m}}$

Q21. If you pull the rubber band back twice as far, use a projectile that is 1/3 the mass, and launch from a height that is half as tall, by what factor will the distance flown by the projectile change?

(2d) $\sqrt{\frac{\frac{1}{2}h}{\frac{1}{3}m}} = 2\sqrt{\frac{3}{2}} * d\sqrt{\frac{h}{m}} = 2.4 * d\sqrt{\frac{h}{m}}$, so the projectile will travel 2.4 times as far.

Equivalently: doubling the distance pulled multiplies by 2, multiplying the mass by 1/3 multiplies the travel distance by $\sqrt{\frac{1}{3}}$, and multiplying the height by 1/2 multiplies the travel distance by $\sqrt{\frac{1}{2}}$. Putting all those together gives $2 * \sqrt{\frac{1}{3}} * \sqrt{\frac{1}{2}} = 2.4$